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Possible Determination of the Spin of the Ξ^- from the Angular Asymmetry of Its Decay*

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The Ξ^- hyperon is produced in a parity-conserving interaction involving only a proton and two spinless K mesons. Therefore, the Ξ^- beam in any direction consists of an equal, incoherent mixture of two pure spin states which go into each other under 180° rotation in the production plane. This condition implies a new restriction upon the angular distribution of the Ξ^- decay products. In particular, the distribution $I(\theta,\phi)=1+A\cos\theta$ is impossible for $A\neq 0$ unless the Ξ^- spin is $\frac{1}{2}$.

1. INTRODUCTION

RECENT experiments[1] suggest the possibility that the usual methods of analyzing angular distributions in the decay of the Ξ^- hyperon may not be able to distinguish between spin $\frac{1}{2}$ and spin $\frac{3}{2}$ for the hyperon. In these experiments, the hyperon is produced in one of the reactions

$$K^-+p\rightarrow\Xi^-+K^+, \tag{1.1}$$

$$K^-+p\rightarrow\Xi^-+K+\pi, \tag{1.2}$$

and subsequently decays according to

$$\Xi^-\rightarrow\Lambda^0+\pi^-. \tag{1.3}$$

The method of Adair[2] suffers from the paucity of Ξ^- production events wherein all particles move nearly along the line of the incident K^- beam. The method of Lee and Yang[3] applies to all Ξ^- directions. However, it results at best in a series of inequalities which cannot exclude Ξ^- spin J unless the measured decay asymmetry satisfies

$$|\langle\cos\theta\rangle|>1/6J \tag{1.4}$$

for some production direction of the Ξ^-. The polar angle θ in relation (1.4) is that formed by the momentum of the decay pion, in the rest system of the hyperon. The polar (z) axis is perpendicular to the plane containing the momenta of the Ξ^- and the incident K^-. This method of analysis involves no assumptions about the production process, and is therefore incapable of making effective use of the azimuthal angular distribution in decay.

The asymmetry theorems of Peshkin[4] also fail to decide between the possible spins of $\frac{1}{2}$ and $\frac{3}{2}$ for the Ξ^-. The failure here is due to discarding the information contained in the odd part of the angular distribution in parity-mixing decay.

The discussion below extends the second asymmetry theorem of reference 4 to cover decay of Ξ^- hyperons produced in reaction (1.1). The argument makes use of the zero spin of the K meson and of parity conservation in the production process. The result is an unambiguous test between spin $\frac{1}{2}$ and spin $\frac{3}{2}$, provided that the decay gives some evidence of parity mixing, and that the decay angular distributions are measured with sufficient accuracy. Thus the right-hand side of inequality (1.4) is replaced by zero.

2. ASYMMETRY CONDITIONS

In reaction (1.1), the initial beam consists of an equal, incoherent mixture of two pure quantum states. In one state the proton spin is parallel to the K^- momentum; in the other it is antiparallel. These two states go into each other under the symmetry operation which consists of space inversion followed by 180° rotation about the normal to the production plane. Under the same symmetry operation, the momentum of the Ξ^- goes into itself. Therefore the Ξ^- "beam" in a given direction consists of an equal, incoherent mixture

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of two pure spin states,[5] represented by ψ_J and $R\psi_J$, where R represents the rotation described above. These two wave functions need not be orthogonal. They may even be identical. In terms of the eigenfunctions $\phi_{J\mu}$ of J_z,

$$\psi_J=\sum_\mu \beta_\mu \phi_{J\mu}, \quad (2.1)$$

where the β_μ are $(2J+1)$ complex numbers obeying

$$\sum_\mu |\beta_\mu|^2=1. \quad (2.2)$$

Except for a phase factor, the rotated spin state is represented by

$$R\psi_J=\sum_\mu(-1)^{J-\mu}\beta_\mu\phi_{J\mu}. \quad (2.3)$$

Upon decay, $\phi_{J\mu}$ takes the form

$$\phi_{J\mu}=p(Y_{J-\frac{1}{2}}\cdot\Lambda_{\frac{1}{2}})_{J\mu}+d(Y_{J+\frac{1}{2}}\cdot\Lambda_{\frac{1}{2}})_{J\mu}. \quad (2.4)$$

The spherical harmonic Y refers to the relative momentum of the π^-, Λ^0 system in the coordinate system described under Eq. (1.4). The spinor $\Lambda_{\frac{1}{2}}$ is a spin wave function for the Λ^0 hyperon. The dot product indicates vector coupling, with the Condon-Shortley phase convention.[6] The complex numbers p and d are normalized so that

$$|p|^2+|d|^2=1. \quad (2.5)$$

They determine the parity-mixing coefficient α through

$$\alpha=p^*d+pd^*. \quad (2.6)$$

It obeys

$$-1\leqslant\alpha\leqslant 1. \quad (2.7)$$

Let the angular distribution $I(\theta,\phi)$ of the decay pions be expressed, in the same coordinate system, as

$$I(\theta,\phi)=\sum_{L,M} a(L,M)Y_{LM}(\theta,\phi), \quad (2.8)$$

$$a(L,M)=(-1)^M a(L,-M)^* = \int Y_{LM}(\theta,\phi)^* I(\theta,\phi)d\Omega, \quad (2.9)$$

$$a(L,M)=\tfrac{1}{2}[\langle\psi_J|Y_{LM}^*|\psi_J\rangle+\langle R\psi_J|Y_{LM}^*|R\psi_J\rangle] =\tfrac{1}{2}[1+(-1)^M]\langle\psi_J|Y_{LM}^*|\psi_J\rangle. \quad (2.10)$$

Thus the angular distribution upon decay is that from a *single quantum state*, except that the odd-M terms are absent. Combining Eqs. (2.1) and (2.10) gives

$$a(L,-M)=\langle\psi_J\|Y_L\|\psi_J\rangle \times\sum_\mu \beta_{\mu+M}^*\beta_\mu C(J,L,J;\mu,M), \quad (2.11)$$

where the last factor is the vector-coupling coefficient.[6] The reduced matrix elements are purely geometric quantities given by

$$\langle\psi_J\|Y_L\|\psi_J\rangle=\left[\frac{(2L+1)(2J+L+2)(2J-L+1)}{4\pi(2J+1)(2J+2)}\right]^{1/2} \times C(J+\tfrac{1}{2},L,J+\tfrac{1}{2};0,0), \quad (2.12)$$

for even $L<2J$, and by

$$\langle\psi_J\|Y_L\|\psi_J\rangle=-\alpha\left[\frac{L(L+1)(2L+1)}{4\pi(2J)(2J+1)}\right]^{1/2} \times C(J+\tfrac{1}{2},L,J-\tfrac{1}{2};0,0), \quad (2.13)$$

for odd $L\leqslant 2J$.

Relations (2.11) constitute a set of restrictions on the asymmetry coefficients $a(L,M)$ for $L\neq 0$, since they must be satisfied by $(2J+1)$ complex numbers β_μ and one real number α in the ranges given by Eqs. (2.2) and (2.7). The restrictions on $a(L,0)$ alone are those given by Lee and Yang.[3] Those for $M>0$ are new and independent. The second asymmetry theorem of reference 4 is implied by, but is not as strong as, the conditions (2.11).

3. REJECTION CRITERION FOR $J=\frac{3}{2}$

When the data are sufficiently numerous to determine all the $a(L,M)$ accurately, the hyperon spin is determined unambiguously, except in one case which is discussed below. Any $a(L,M)$ different from zero for $L>1$ excludes $J=\frac{1}{2}$. Vanishing of $a(L,M)$ for all $L>1$ excludes[4] $J>\frac{3}{2}$. We now consider the most interesting case, $J=\frac{3}{2}$, in detail.

Equations (2.11) for $M=0$ are just the conditions of Lee and Yang,[3] which they express in the convenient form

$$\begin{aligned} I_{\frac{3}{2}}+I_{-\frac{3}{2}}&=(1/2)-(5/2)\langle P_2\rangle,\\ I_{\frac{1}{2}}+I_{-\frac{1}{2}}&=(1/2)+(5/2)\langle P_2\rangle,\\ \alpha(I_{\frac{3}{2}}-I_{-\frac{3}{2}})&=-(9/2)\langle P_1\rangle+(7/6)\langle P_3\rangle,\\ \alpha(I_{\frac{1}{2}}-I_{-\frac{1}{2}})&=-(3/2)\langle P_1\rangle-(7/2)\langle P_3\rangle, \end{aligned} \quad (3.1)$$

where

$$I_\mu=|\beta_\mu|^2, \quad (3.2)$$

$$\langle P_L\rangle=(-1)^L[4\pi/(2L+1)]^{1/2}a(L,0). \quad (3.3)$$

The two Eqs. (2.11) for $M=2$ are conveniently combined to give the new condition

$$\alpha^2(I_{\frac{3}{2}}I_{-\frac{1}{2}}+I_{\frac{1}{2}}I_{-\frac{3}{2}})=\alpha^2(25/48)|\langle P_{22}\rangle|^2 +(49/432)|\langle P_{32}\rangle|^2, \quad (3.4)$$

where

$$\langle P_{L2}\rangle=(-1)^L\left[\frac{4\pi}{2L+1}\frac{(L+2)!}{(L-2)!}\right]^{1/2}a(L,2). \quad (3.5)$$

Now it is easy to see that if all the asymmetry coefficients for $L>1$ vanish, then Eq. (3.4) contradicts Eqs. (3.1) unless $\alpha=0$, in which case $\langle P_1\rangle$ must also vanish. Therefore, any set of accurate data decide

[5] The usefulness of a limited number of pure states was pointed out by P. Eberhard and M. L. Good, Phys. Rev. **120**, 1442 (1960).
[6] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957).

unambiguously between $J=\frac{1}{2}$ and $J=\frac{3}{2}$, unless the decay is isotropic. Isotropic decay also excludes $J=\frac{3}{2}$ if α is known from other evidence to differ from zero.

4. FINITE SAMPLES

When the measured $\langle P\rangle$ are subject to statistical errors, the asymmetry conditions exclude $J=\frac{3}{2}$ with large probability unless there is a satisfactory set α, I_μ which satisfies Eqs. (3.1) and (3.4) with reasonable probability. Although the $\langle P_L\rangle$ depend upon the momentum of the Ξ^-, it is possible to average all decay events in testing Eqs. (3.1), because the equations are linear in the $\langle P_L\rangle$ and α is fixed. Even production events from reaction (1.2) may be included, since the assumption of only two quantum states is not involved in Eqs. (3.1). In practice, a more severe test is obtained from a separate treatment of the Ξ^- produced in different cones about the axis of the K^- beam, so chosen that each $\langle P_L\rangle$ is nearly constant in each cone.

However, if the hypothesis $J=\frac{3}{2}$ "passes" the test (3.1), then the data must be divided into separate cones to perform the test (3.4). For typical possible values of α, one must solve Eqs. (3.1) for the I_μ and their correlated errors, using only data from reaction (1.1). Then if, for all values of α, the measured $|\langle P_{L2}\rangle|^2$ fail the test with high probability for some cone, or if they pass with only fair probability for each cone, $J=\frac{3}{2}$ is excluded with high probability.

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Nuclear Charge Distributions in the Interaction of 2.9-GeV Protons with Heavy Elements*

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Cross sections for the formation of a number of nuclides in the mass range 65–74 in the bombardment of In, Au, and U with 2.9-GeV protons have been measured. Four isobars of mass 72, three of mass 67, and three of mass 66 are included. Charge distribution curves have been constructed from the data, using N/Z the ratio of neutrons to protons in the nucleus, as abscissa. The curves are not symmetric about the peak, falling less steeply toward large N/Z, with U having the most asymmetry. The peak position shifts to larger N/Z as the target mass increases. The data for In are consistent with a cascade-evaporation mechanism involving a long evaporation sequence, while the U data show the importance of low-excitation-energy fission in the formation of neutron-excess nuclides.

INTRODUCTION

RADIOCHEMICAL studies of the interaction of protons of energy in the GeV range with heavy elements (uranium,[1–4] lead,[3,5] and tantalum[6]) have led to the following general description of the mass-yield curve. The cross section for forming nuclides of a given mass number decreases as the mass number decreases, with no indication of any prominent fission peak, until low mass numbers are reached, where the cross section rises with decreasing mass number. There is a region of intermediate masses where the cross section is approximately constant. This behavior is in contrast to that observed at lower energies, where two distinct regions corresponding to spallation and fission are separated by a region of very low cross sections. It is evident from emulsion studies[7,8] that fission occurs to an appreciable extent at GeV energies, even with nuclei as light as Ag and Br. The problem is to determine the relation, if any, of the cross sections observed at these energies to the concepts of fission and spallation which have been useful at lower energies. A third process, fragmentation, which is a specifically high-excitation-energy process, has been postulated to account for the yields and excitation functions of nuclides with mass number less than about 40.

Previous work has shown that nuclides on both sides of the beta stability line are formed in appreciable amounts, and that stable nuclei, which are usually not

* Research performed under the auspices of the U. S. Atomic Energy Commission.
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